KODAK GRAY SCALE

C	Red-Filter Negative	Cyan Printer	M	Green-Filter Negative	Magenta Printer	Y	Blue-Filter Negative	Yellow Printer

.00 A .10 .20 .30 .50 .70 M 1.00 1.30 1.60 B 1.90

black	3-color	white	cyan	violet	magenta	primary red	yellow	green

KODAK COLOR CONTROL PATCHES

These colors have been selected as representative of those inks commonly used in photomechanical reproduction.

VENERABILI ET PRAECLARO

SENI

JOANNI CHRISTIANO LUDOVICO HELLWIGIO

A. A. L. L. M. DUCIS BRUNSVICENS. LUNEBURGENS. AB AULAE CONSILIIS, COLLEGII CAROLINI MATHESEOS ET HISTORIAE NATURALIS P. P. O. ET SOCIETT. LITERARR. PLURR. SOC.

NATALITIA OCTOGESIMA QUARTA

GRATULATURUS

NONNULLARUM SERIERUM INFINITARUM EVOLUTIONEM NOVAM

PROPOSUIT

FRIDERICUS GUILIELMUS SPEHR,

A. A. L. L. M. DUC. BRUNSVIC. ET LUNEBURGENS. COLLEGII CAROLINI P. P. E. ET SOCIETT. LITERARR. PLURR. SOC.

BRUNSVIGAE

DIE VIII NOV. MDCCCXXVII.

8. 11. 1827

TYPIS FR. VIEWEGII ET FILII.

§. 1.

In tractatu de analyseos recentioris determinandi utraque ratione *) ostendi, quomodo inductionis ratiocinatione generaliter utamur; hoc autem loco in casibus specialibus majorem generalitatem esse ponendam mihi liceat demonstrare. Quod si evenerit factores constantes omnibus in terminis scalae recursionis fingendi sunt, ita ut formula in eodem maneat. Exstant e. gr. series nonnullae infinitae, in quarum evolutione ad hoc respici necesse est, series infinita pro $\mathrm{Cos}\, x$, $(1 + a)^x$ etc., quas hic exhibebo.

I. Series infinita pro Cos x.

§. 2.

Licet autem pro Cos x seriem infinitam secundum potentias positivas et integras ips. x progredientem ponere, ita ut habeamus:

$$\mathrm{Cos}\, x = \overset{0}{B} + \overset{1}{B}x + \overset{2}{B}x^2 + \overset{3}{B}x^3 \ldots + \overset{2n}{B}x^{2n} + \overset{2n+1}{B}x^{2n+1} + \ldots$$

---

*) Brunsv. 1824.

pro $x=o$ est $\operatorname{Cos} x=1$, ex quo sequitur, esse $\overset{0}{B}=1$. Valore pro x eodem sed negativo substituto, erit:

$$\operatorname{Cos}(-x)=1-\overset{1}{B}x+\overset{2}{B}x^2-\overset{3}{B}x^3\ldots+\overset{2n}{B}x^{2n}-\overset{2n+1}{B}x^{2n+1}+\ldots$$

Cum autem $\operatorname{Cos}(-x)=\operatorname{Cos} x$, adest $\overset{1}{B}=o$, $\overset{3}{B}=o$, et generaliter $\overset{2n+1}{B}=o$, ita ut habeatur:

$$\operatorname{Cos} x = 1+\overset{2}{B}x^2+\overset{4}{B}x^4+\ldots+\overset{2n}{B}x^{2n}\ldots$$

seu signis simplicioribus assumtis:

$$\operatorname{Cos} x = 1+\overset{1}{A}x^2+\overset{2}{A}x^4+\ldots+\overset{n}{A}x^{2n}\ldots$$

ejus seriei coefficientes adhuc deducendi sunt. Habetur autem:

$$(\operatorname{Cos} x)^2 = 1+2\overset{1}{A}x^2+(2\overset{2}{A}+\overset{1}{A}{}^2)x^4+\ldots$$
$$+(\overset{n}{A}+\overset{n-1}{A}\overset{1}{A}+\overset{n-2}{A}\overset{2}{A}+\ldots\overset{n-4}{A}\overset{4}{A}+\ldots\overset{1}{A}\overset{n-1}{A}+\overset{n}{A})x^{2n}+\ldots$$

Notum est, relationem exstare sequentem:

$$(\operatorname{Cos} x)^2 = \frac{1+\operatorname{Cos} 2x}{2},$$

quare, si seriem positam abhibeamus, invenietur:

$$\operatorname{Cos} 2x = 1+2^2\overset{1}{A}x^2+2^4\overset{2}{A}x^4\ldots+2^{2n}\overset{n}{A}x^{2n}\ldots$$

ex quo satis elucet, esse generaliter:

$$(\operatorname{Cos} x)^2 = \tfrac{1}{2}(2^1+2^2\overset{1}{A}x^2+2^4\overset{2}{A}x^4+\ldots+2^{2n}\overset{n}{A}x^{2n}\ldots)$$
$$= 1+2\overset{1}{A}x^2+2^3\overset{2}{A}x^4+\ldots+2^{2n-1}\overset{n}{A}x^{2n}\ldots$$

Hinc autem concluditur poni posse:

$$2^{2n-1}\overset{n}{A} = \overset{n}{A}+\overset{n-1}{A}\overset{1}{A}+\overset{n-2}{A}\overset{2}{A}\ldots+\overset{n-h}{A}\overset{h}{A}\ldots+\overset{1}{A}\overset{n-1}{A}+\overset{n}{A}$$

sive:

$$(2^{2n-1}-2)\overset{n}{A} = \overset{n-1}{A}\overset{1}{A} + \overset{n-2}{A}\overset{2}{A} \ldots + \overset{n-h}{A}\overset{h}{A} \ldots + \overset{1}{A}\overset{n-1}{A}.$$

Inter quantitates $\overset{n}{A}$, $\overset{n-1}{A}$, $\overset{n-2}{A}$,... habetur ergo recursio sequens:

$$\overset{n}{A} = \frac{\overset{n-1}{A}\overset{1}{A} + \overset{n-2}{A}\overset{2}{A} + \ldots + \overset{n-h}{A}\overset{h}{A} \ldots + \overset{1}{A}\overset{n-1}{A}}{2^{2n-1}-2}$$

§. 3.

Coefficientes binomiales n^{tae} potentiae, qui per ${}^{n}\overset{0}{\mathfrak{B}}$, ${}^{n}\overset{1}{\mathfrak{B}}$, ${}^{n}\overset{2}{\mathfrak{B}}$,... ${}^{n}\overset{h}{\mathfrak{B}}$,... indicentur sequentem relationem praebent:

$$2^n = {}^{n}\overset{0}{\mathfrak{B}} + {}^{n}\overset{1}{\mathfrak{B}} + {}^{n}\overset{2}{\mathfrak{B}} \ldots + {}^{n}\overset{h}{\mathfrak{B}} \ldots + {}^{n}\overset{n}{\mathfrak{B}}$$

seu:

$$2^n - 2 = {}^{n}\overset{1}{\mathfrak{B}} + {}^{n}\overset{2}{\mathfrak{B}} \ldots + {}^{n}\overset{h}{\mathfrak{B}} \ldots + {}^{n}\overset{n-1}{\mathfrak{B}}$$

Significetur per $\overset{n}{P}$ numerus permutationis n^{ti} gradus, et sit:

$$\frac{1}{\overset{n}{P}} = \overset{-n}{P}$$

adest, si in utramque partem aequationis pro 2^n-2 per $\overset{-n}{P}$ multiplicetur:

$$\overset{-n}{P}(2^n-2) = \overset{-(n-1)}{P}\overset{-1}{P} + \overset{-(n-2)}{P}\overset{-2}{P} + \ldots + \overset{-(n-h)}{P}\overset{-h}{P} \ldots + \overset{-1}{P}\overset{-(n-1)}{P}$$

vel pro n valor $2n-1$ si ponatur, evenit:

$$\overset{-(2n-1)}{P} = \frac{\overset{-(2n-2)}{P}\overset{-1}{P} + \overset{-(2n-3)}{P}\overset{-2}{P} + \ldots + \overset{-(2n-1-h)}{P}\overset{-h}{P} \ldots + \overset{-1}{P}\overset{-(2n-2)}{P}}{2^{2n-1}-2}$$

§. 4.

Recursio modo inventa cum ea pro $\overset{n}{A}$ accuratissime convenit, quare colligi possit, esse:

$$\overset{n}{A} = \overset{-(2n-1+2k)}{P}$$

k denotante numero arbitrario et integro. Constat autem, coefficientes seriei ipsius Cos x aliam habere formam. Attamen satis elucet, recursionem pro $\overset{-(2n-1)}{P}$ eandem manere, si pro $\overset{-(2n-1)}{P}$ substituatur $\overset{-(2n-1)}{P} a^{2n-1}$, a denotante numero arbitrario; in utramque enim partem si per a^{2n-1} multiplicetur, evenit:

$$\overset{2n-1}{P} a^{2n-1} = \frac{\overset{-(2n-2)}{P}\overset{-1}{P} a^{2n-1} + \overset{-(2n-3)}{P}\overset{-2}{P} a^{2n-1} \dots + \overset{-(2n-1-h)}{P}\overset{-h}{P} a^{2n-1} \dots + \overset{-1}{P}\overset{-(2n-2)}{P} a^{2n-1}}{2^{2n-1}-2}$$

In termino vero generali potentiam a^{2n-1} in $a^{2n-1-h} \times a^h$ transponere licet, quare habetur:

$$\overset{2n-1}{P} a^{2n-1} = \frac{\overset{-(2n-2)}{P} a^{2n-2} \overset{-1}{P}{}^{1} a + \overset{-(2n-3)}{P} a^{2n-3} \overset{-2}{P} a^2 + \dots + \overset{-(2n-1-h)}{P} a^{2n-1-h} \overset{-h}{P} a^h \dots + \overset{-1}{P} a^1 \overset{-(2n-2)}{P} a^{2n-2}}{2^{2n-1}-2}$$

seu si simpliciter pro $\overset{-(2n-1)}{P} a^{2n-1}$, numero tantum ab n dependente, ponatur $\overset{n}{K}$:

$$\overset{n}{K} = \frac{\overset{n-1}{K}\overset{1}{K} + \overset{n-2}{K}\overset{2}{K} + \overset{n-3}{K}\overset{3}{K} \dots + \overset{n-h}{K}\overset{h}{K} \dots + \overset{1}{K}\overset{n-1}{K}}{2^{2n-1}-2}$$

qua ex formula recursionis, eam ipsam recursioni pro $\overset{n}{A}$ (§ 2) accuratissime congruere, satis patet.

His igitur peractis manifestum est, poni posse:

$$\overset{n}{A} = \overset{n+k}{K}$$

k exprimente quantitate constante, i. e.

$$\overset{n}{A} = \overset{-(2n-1+2k)}{P} a^{2n-1+2k}$$

Superest autem, ut ambae constantes, a et k, determinentur. Ad quod efficiendum substituatur pro n valor 1, adest $\overset{n}{A} = \overset{1}{A}$, habetur ergo: $\overset{1}{A} = \overset{-(1+2k)}{P} a^{1+2k}$

Ex aliis autem argumentis demonstratur, esse:

$$\overset{1}{A} = -\frac{1}{1.2} = -\overset{-2}{P}$$

quare fit:

$$\overset{-(2k+1)}{P} a^{2k+1} = -\overset{-2}{P}$$

Constantes k et a ita ergo determinandi sunt, ut huic aequationi satisfaciant; manifestum est autem, fieri: $1+2k=2$ ergo $k=\frac{1}{2}$ quo posito habemus:

$$\overset{-2}{P} a^2 = -\overset{-2}{P}$$

ex quo denique colligitur: $a^2=1$, et $a=\sqrt{-1}$, et generaliter:

$$\overset{n}{A} = (\sqrt{-1})^{2n}\overset{-2n}{P} = (-1)^n\overset{2n}{P}$$

Series igitur pro Cos x in sequenti forma videtur:

$$\operatorname{Cos} x = 1 - \overset{-2}{P}x^2 + \overset{-4}{P}x^4 - \overset{-6}{P}x^6 \ldots + (-1)^n\overset{-2n}{P}x^{2n} \ldots$$

ut satis notum est.

II. Series infinita pro Sin x.

§ 5.

Quemadmodum pro Cos x seriem secundum potentias positivas et integras ipsius x progredientem substituimus, ita hic poni potest generaliter:

$$\operatorname{Sin} x = \overset{0}{B} + \overset{1}{B}x + \overset{2}{B}x^2 + \overset{3}{B}x^3 \ldots + \overset{2n}{B}x^{2n} + \overset{2n+1}{B}x^{2n+1} \ldots.$$

pro $x=0$ adest $\operatorname{Sin} x=0$ ex quo fit $\overset{0}{B}=0$; item pro x valore $-x$ accepto invenitur:

$$\operatorname{Sin}(-x) = -\overset{1}{B}x + \overset{2}{B}x^2 - \overset{3}{B}x^3 \ldots, + \overset{2n}{B}x^{2n} - \overset{2n+1}{B}x^{2n+1} \ldots$$

Notum autem est constare:

$$\mathrm{Sin}(-x) = -\mathrm{Sin}\,x$$

i. e.

$$\mathrm{Sin}(-x) = -\overset{1}{B}x - \overset{2}{B}x^2 - \overset{3}{B}x^3 - \ldots - \overset{2n}{B}x^{2n} - \overset{2n+1}{B}x^{2n+1}$$

quare habetur

$$\overset{2}{B} = o,\ \overset{4}{B} = o \text{ et generaliter } \overset{2n}{B} = o$$

ita ut series invenienda sequentem habeat formam:

$$\mathrm{Sin}\,x = \overset{1}{B}x + \overset{3}{B}x^3 + \overset{5}{B}x^5 \ldots + \overset{2n+1}{B}x^{2n+1} \ldots$$

sive si signis simplicioribus utamur:

$$\mathrm{Sin}\,x = \overset{0}{A}x + \overset{1}{A}x^3 + \overset{2}{A}x^5 \ldots + \overset{n}{A}x^{2n+1} \ldots$$

Cum autem, ut notum:

$$\mathrm{Sin}\,2x = 2\mathrm{Sin}\,x\ \mathrm{Cos}\,x$$

et series pro Cos x jam reperta sit, primum hanc seriem cum cognita pro 2Cos x multiplicando, deinde in serie ficta ipsius Sin x pro x valorem 2x substituendo, duae series aequales invenientur, in quibus terminos aequealtos esse aequales oportet. Ponatur ergo:

$$\mathrm{Sin}\,x = \overset{0}{A}x + \overset{1}{A}x^3 + \overset{2}{A}x^5 \ldots + \overset{n}{A}x^{2n+1} \ldots$$

$$2\mathrm{Cos}\,x = 2(1 - \overset{-2}{P}x^2 + \overset{-4}{P}x^4 \ldots + (-1)^n \overset{-2n}{P}x^{2n} \ldots)$$

fit:

$$\mathrm{Sin}\,2x = 2\overset{0}{A}x + 2(\overset{1}{A} - \overset{-2}{P}\overset{0}{A})x^3 + 2(\overset{2}{A} - \overset{-2}{P}\overset{2}{A} + \overset{4}{P}\overset{0}{A})x^5 \ldots$$

$$\ldots + 2(\overset{n}{A} - \overset{-2}{P}\overset{n-1}{A} + \overset{-4}{P}\overset{n-2}{A} - \ldots + (-1)^h \overset{-2h}{P}\overset{n-h}{A} \ldots (-1)^n \overset{-2n}{P}\overset{0}{A})x^{2n+1} \ldots$$

Cum autem:

$$\mathrm{Sin}\,2x = 2\overset{0}{A}x + 2^3\overset{1}{A}x^3 + 2^5\overset{2}{A}x^5 \ldots + 2^{2n+1}\overset{n}{A}x^{2n+1} \ldots$$

habetur generaliter:

$$2^{2n+1}\overset{n}{A} = 2(\overset{n}{A} - \overset{-2}{P}\overset{n-1}{A} + \overset{-4}{P}\overset{n-2}{A} - \ldots + (-1)^h \overset{2h}{P}\overset{n-h}{A} \ldots (-1)^n \overset{2n}{P}\overset{0}{A})$$

$$(2^{2n}-1)\overset{n}{A} = -\overset{-2}{P}\overset{n-1}{A} + \overset{-4}{P}\overset{n-2}{A} - \ldots + (-1)^h \overset{2h}{P}\overset{n-h}{A} \ldots (-1)^n \overset{2n}{P}\overset{0}{A}$$

sive:

$$(-1)^n\overset{n}{A} = \frac{\overset{-2}{P}(-1)^{n-1}\overset{n-1}{A} + \overset{-4}{P}(-1)^{n-2}\overset{n-2}{A} + \ldots + \overset{-2h}{P}(-1)^{n-h}\overset{n-h}{A} \ldots + \overset{2n}{P}(-1)^{0}\overset{0}{A}}{2^{2n}-1}$$

§ 6.

In § 3 derivata est recursio sequens:

$$\overset{-n}{P}(2^n-2) = \overset{-(n-1)}{P}\overset{-1}{P} + \overset{-(n-2)}{P}\overset{2}{P} + \ldots + \overset{-(n-2h)}{P}\overset{2h}{P} + \overset{-(n-(2h+1))}{P}\overset{-(2h+1)}{P} \ldots + \overset{-1}{P}\overset{-(n-1)}{P}.$$

vel si pro n valor $2n+1$ substituatur:

$$\overset{-(2n+1)}{P}(2^{2n+1}-2) = \overset{-2n}{P}\overset{-1}{P} + \overset{-(2n-1)}{P}\overset{-2}{P} + \overset{-(2n-2)}{P}\overset{-3}{P} \ldots + \overset{-(2n-(2h-1))}{P}\overset{-2h}{P} + \overset{-(2n-2h)}{P}\overset{-(2h+1)}{P} \ldots + \overset{-2}{P}\overset{-(2n-1)}{P} + \overset{-1}{P}\overset{-2n}{P} \quad \odot$$

item ex proprietatibus coefficientium binomialium patet, esse:

$$0 = 1 - {}^{2n+1}\overset{1}{\mathfrak{B}} + {}^{2n+1}\overset{2}{\mathfrak{B}} - \ldots + {}^{2n+1}\overset{2h}{\mathfrak{B}} - {}^{2n+1}\overset{2h+1}{\mathfrak{B}} \ldots + {}^{2n+1}\overset{2n}{\mathfrak{B}} - {}^{2h+1}\overset{2n+1}{\mathfrak{B}}$$

qua ex relatione si in utramque partem per $\overset{-(2n+1)}{P}$ multiplicetur, habemus:

$$0 = \overset{-(2n+1)}{P} - \overset{-2n}{P}\overset{-1}{P} + \overset{-(2n-1)}{P}\overset{-2}{P} - \overset{-(2n-2)}{P}\overset{-3}{P} \ldots + \overset{-(2n-(2h-1))}{P}\overset{-2h}{P} - \overset{-(2n-2h)}{P}\overset{-(2h+1)}{P} \ldots + \overset{-2n}{P}\overset{-1}{P} - \overset{-(2n+1)}{P}$$

quo ad ($\odot$) addito dabit:

$$\overset{-(2n+1)}{P}(2^{2n+1}-2) = 2(\overset{-(2n-1)}{P}\overset{-2}{P} + \overset{-(2n-3)}{P}\overset{-4}{P} + \ldots + \overset{-(2n-(2h-1))}{P}\overset{-2h}{P} \ldots + \overset{-1}{P}\overset{-2n}{P})$$

seu:

$$\overset{-(2n+1)}{P} = \frac{\overset{-(2n-1)}{P}\overset{-2}{P} + \overset{-(2n-3)}{P}\overset{-4}{P} + \ldots, + \overset{-(2n-(2h-1))}{P}\overset{-2h}{P} \ldots + \overset{-1}{P}\overset{-2n}{P}}{2^{2n}-1}$$

et si pro $\overset{-(2n+1)}{P}$, numero tantum ab n dependente, signum $\overset{n}{C}$ introducatur, evenit:

$$\overset{n}{C} = \frac{\overset{n-1}{C}\overset{-2}{P} + \overset{n-2}{C}\overset{-4}{P} + \ldots + \overset{n-h}{C}\overset{2h}{P} \ldots + \overset{0}{C}\overset{-2n}{P}}{2^{2n}-1}$$

Qua ex formula recursionis satis elucet, *factores constantes arbitrarios* poni non posse, quia in hac formula ii, qui rationem et modum recursionis constituunt, numeri jam determinati, scilicet numeri permutationis $\overset{-2}{P}$, $\overset{-4}{P}$, $\overset{-6}{P}$... quorum exponentes sunt invariabiles. Derivationem hujus seriei ad intelligendum hoc discrimen adhibui. Habetur ergo:

$$(-1)^n \overset{n}{A} = \overset{n+k}{C} = \overset{-(2n+2k+1)}{P}$$

Pro $n = 0$ erit $\overset{n}{A} = \overset{0}{A} = 1$, quod ex aliis argumentis patet, adest igitur:

$$1 = \overset{-(2k+1)}{P}$$

ex quo sequitur, esse $k = 0$ et generaliter reperietur

$$\overset{n}{A} = (-1)^n \overset{-(2n+1)}{P}$$

sic, ut series pro Sin x sequentem habeat formam:

$$\operatorname{Sin} x = x - \overset{-3}{P} x^3 + \overset{-5}{P} x^5 \ldots + (-1)^n \overset{-(2n+1)}{P} x^{2n+1} + \ldots$$

III. Series infinita pro $(1 + a)^x$

§ 7.

Exemplum factoribus constantibus ponendis conveniens evolutio functionis transcendentis $(1 + a)^x$ praebet. Sumta serie functioni huic aequali secundum potentias positivas et integras ipsius x progrediente, apparet, terminum initialem esse $= 1$, quia pro $x = 0$ erit $(1 + a)^x = 1$, habetur ergo:

$$(1 + a)^x = 1 + \overset{1}{A} x + \overset{2}{A} x^2 + \overset{3}{A} x^3 \ldots + \overset{n}{A} x^n + \ldots$$

et, si in utramque partem ad quadratum elevetur:

$$(1+a)^{2x} = 1 + 2\overset{1}{A}x + (2\overset{2}{A} + \overset{1}{A}{}^2)x^2 + \ldots.$$
$$+ (\overset{n}{A} + \overset{n-1}{A}\overset{1}{A} + \overset{n-2}{A}\overset{2}{A} + \ldots + \overset{n-h}{A}\overset{h}{A} \ldots + \overset{1}{A}\overset{n-1}{A} + \overset{n}{A})x^n \ldots.$$

Attamen functio $(1+a)^{2x}$, in serie ficta pro $(1+a)^x$ valorem $2x$ in loco ipsius x ponendo, in aliam quoque seriem infinitam evolvi potest:

$$(1+a)^{2x} = 1 + 2\overset{1}{A}x + 2^2\overset{2}{A}x^2 + 2^n\overset{n}{A}x^n \ldots$$

ex quo colligitur, esse generaliter:

$$2^n\overset{n}{A} = \overset{n}{A} + \overset{n-1}{A}\overset{1}{A} + \overset{n-2}{A}\overset{2}{A} \ldots + \overset{n-h}{A}\overset{h}{A} + ,\ldots + \overset{1}{A}\overset{n-1}{A} + \overset{n}{A}$$
$$(2^n - 2)\overset{n}{A} = \overset{n-1}{A}\overset{1}{A} + \overset{n-2}{A}\overset{2}{A} \ldots + \overset{n-h}{A}\overset{h}{A} + \ldots + \overset{1}{A}\overset{n-1}{A}$$

et:

$$\overset{n}{A} = \frac{\overset{n-1}{A}\overset{1}{A} + \overset{n-2}{A}\overset{2}{A} + \ldots + \overset{n-h}{A}\overset{h}{A} + \ldots + \overset{1}{A}\overset{n-1}{A}}{2^n - 2}$$

§ 8.

In § 3 jam expositum est, inter numeros permutationis negativis cum exponentibus sequentem recursionis formulam existere:

$$\overset{-n}{P} = \frac{\overset{-(n-1)}{P}\overset{-1}{P} + \overset{-(n-2)}{P}\overset{-2}{P} + \ldots + \overset{(-h)}{P}\overset{-h}{P} \ldots \overset{-1}{P}\overset{n-(n-1)}{P}}{2^n - 2}$$

in utramque ergo partem par a^n multiplicando, lit. a denotante numero arbitrario coustante, habetur:

$$\overset{-n}{P}a^n = \frac{\overset{-(n-1)}{P}a^{n-1}\overset{-1}{P}a^1 + \overset{-(n-2)}{P}a^{n-2}\overset{-2}{P}a^2 \ldots + \overset{-(n-h)}{P}a^{n-h}\overset{-h}{P}a^h \ldots + \overset{-1}{P}a\overset{-(n-1)}{P}a^{n-1}}{2^n - 2}$$

vel si generaliter $\overset{-n}{P}a^n$ per $\overset{n}{M}$ exprimatur,

$$\overset{n}{M} = \frac{\overset{n-1}{M}\overset{1}{M} + \overset{n-2}{M}\overset{2}{M} + \ldots + \overset{n-h}{M}\overset{h}{M} \ldots + \overset{1}{M}\overset{n-1}{M}}{2^n - 2}$$

ex quo, ratiocinatione identitatis duarum recursionum adhibita, esse:

$$\overset{n}{A} = \overset{n+k}{M} = \overset{-(n+k)}{P} a^{n+k}$$

colligitur. Ad constantes autem a et k rite determinandos ponatur $n = 1$, adest $\overset{n}{A} = \overset{1}{A}$, et:

$$\overset{-(1+k)}{P} a^{1+k} = \overset{-1}{P}\overset{1}{A}$$

sequitur ergo, fieri

$$k = o \quad \text{et} \quad a = \overset{1}{A},$$

ita ut generaliter sit:

$$\overset{n}{A} = \overset{-n}{P} A^n$$

et functio $(1 + a)^x$ in sequentem seriem infinitam secundum potentias ips. x progredientem:

$$1 + \overset{-1}{P} A x + \overset{-2}{P} A^2 x^2 + \ldots + \overset{-n}{P} A^n x^n \ldots$$

evolvi possit, ubi coefficiens A, modulus baseos $(1 + a)$, tantum ab a dependit, et per seriem cognitam:

$$a - \frac{a^2}{2} + \frac{a^3}{3} - \ldots + (-1)^{n-1} \frac{a^n}{n} \ldots$$

computatur.

Superest ut ad TE me convertam, SENEX VENERABILIS, cujus natalitia, quantum quidem in me fuit, superioribus illis celebranda mihi sumsi.

Uti scholae complures communis patriae nostrae Germaniae, in quibus *Portensem* satis habeo nominasse, jam dudum hanc laudem habent praecipuam, ut ex earum sinu philologorum accurata doctrina praestantissimorum totae acies prodierint, instar Minervae summi Jovis ex capite prognatae, sic alio in genere *Collegium* nostrum *Carolinum* suam sibi propriamque laudem haud sane prae illa, si quid video, contemnendam, obtinet et tuetur. Quis enim est paulo humanior, ad quem non fama certe et auditione tot clarissimorum mathematicorum nomina pervenerint, qui prima disciplinae suae rudimenta in *Collegio Carolino* posuerunt. Satis est ciere *Joannem Josephum Ide,* qui quamquam litteris praematura morte ereptus est, nihilominus de mathesi egregie meruit; *Carolum Brandanum Mollweide,* qui nuper non sine magno literarum detrimento Lipsiae obiit; *Carolum Fridericum Gaussium* illum, de quo nec quomodo nec quid dicam, satis exploratum habeo, ingens haud dubie decus nostratium. Adde *Conradum Dietericum Stahl,* professorem quondam matheseos apud Landshutenses, nunc Academiae novae Monacensis lumen, *Bartelsium,* Professorem Dorpatensem, alios.

Atque ita *Collegium Carolinum,* utilissimum opus sapientissimorum Ducum nostrorum, CAROLI, qui condidit, CAROLI GUILIELMI FERDINANDI, qui amplificavit et ornavit, FRIDERICI GUILIELMI, qui foede corruptum restituit ad pristinam suam integritatem, CAROLI denique Ducis Serenissimi, qui nunc cum maxime felicissimis auspiciis suis sustentat, et tuetur a maligna invidorum plebecula, literatorum, illiteratorum, vel in hoc uno literarum genere, nam de reliquis non meum est judicare, assignatas sibi partes egregie et administravit et, nisi quod infaustum nos afflet sidus — quod abominamur — in posterum administrabit. Sed ut redeam ad splendidam illam summorum mathematicorum seriem, quorum claritudinis partem aliquam suo sibi jure vindicat *Collegium Carolinum*, Tu Vir summe venerabilis, praecipuus illis auctor fuisti laudis, quam postea sunt consecuti. Tibi ergo et ipsum *Collegium Carolinum* suae existimationis magnam partem refert acceptam et inprimis sibi ducit honorificum, quod Te summa etiam in senectute adhuc suo potitur lumine atque ornamento. Me vero data horum solennium opportunitate benigna quaeso audientia exceperis, interpretem communis omnium et docentium et discentium, quotquot sunt Collegii Carolini, reverentiae, laetitiae, pietatis.

KODAK GRAY SCALE

C Red-Filter Negative Cyan Printer **M** Green-Filter Negative Magenta Printer **Y** Blue-Filter Negative Yellow Printer

.00 A .10 .20 .30 .50 .70 M 1.00 1.30 1.60 B 1.90

black 3-color white cyan violet magenta primary red yellow green

KODAK COLOR CONTROL PATCHES

These colors have been selected as representative of those inks commonly used in photomechanical reproduction.